Analysis of the Collateralized Debt Obligation Scheme
Applied to the Newsboy Problem

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ABSTRACT

As a sequel to the original paper by the same authors, Isogai, Ohashi, and Sumita [5], this paper examines the effect of the collateralized debt obligation (CDO) scheme applied to the classical newsboy problem (NBP). The distribution function of the profit with CDO is derived explicitly as a function of the order quantity $Q$. Sufficient conditions are established under which the optimal solution for the value at risk (VaR) problem with CDO is superior or inferior to that without CDO. Furthermore, the VaR problem of NBP without CDO is analyzed in detail for the case of the exponentially distributed demand, deriving the optimal solution $Q_{NBP}^*$ and $\eta_{NBP}^*$ explicitly. Assuming that the stochastic demand $D$ is exponentially distributed, extensive numerical experiments reveal that the overall effect of CDO is present when the underlying risk for the opportunity loss is rather large.

Keywords: Collateralized debt obligation, risk control, newsboy problem, value at risk
1. INTRODUCTION

It is widely believed that collateralized debt obligations (CDOs) played a major role in the global financial crisis triggered by the collapse of the subprime loan market in the United States. The resulting financial crisis spread all over the world in 2008, throwing the entire world economy into chaos. Usually, the ordinary home loan is designed with a fixed interest rate over a rather long period. In contrast, the subprime loan, which is a home loan provided to low-income people, often involves variable interest rates where fairly low interest rates are applied for the first few years, followed by a sudden increase (typically, the market interest rate plus 3% to 6%) thereafter. If the home price continues to rise, there would be a good chance of refinancing the subprime loan at the time of payment reset so as to avoid this sudden increase in the interest rate. Because of this structure, the subprime loan became popular gradually throughout 1990s.

Around 2003, many financial institutions began to securitize the subprime loans as mortgage-backed securities (MBSs), thereby earning substantial commissions. The expansion of the subprime loan market was accelerated when MBSs for the subprime loans were re-securitized through the means of CDOs, where the resulting financial products were purchased in huge quantity by many investors and financial institutions all over the world. As Barnett-Hart [1] point out, CDO issuance grew from an estimated $20 billion in Q1 2004 to $180 billion by Q1 2007. The driving force behind this boom was the increase in the level of subprime and other non-prime mortgage debt, resulting in the rapid expansion of CDO assets. When the defaults of the subprime loans became visible, home prices began to drop, which in turn triggered other defaults. Soon, the world economy was overwhelmed by this death spiral of subprime loans.

As a consequence, the CDO became notorious for its role in destroying the world economy. However, the fact that the misuse of the CDO resulted in the collapse of the world economy does not necessarily imply that the CDO itself is useless or even hazardous. It is still worth asking whether the CDO would be a genuine financial tool for managing risks.

In the previous paper by the same authors, Isogai, Ohashi, and Sumita [5], the CDO scheme was incorporated by specifying a CDO tranche consisting of a pair of an attachment point $K_a$ and a detachment point $K_d$, as well as the risk-neutral premium $\xi$. Here, the attachment point $K_a$ means that the protection buyer, which is the retailer issuing the CDO, is fully responsible for the opportunity loss up to $K_a$. The protection seller, which is the tranche investor
buying the CDO, compensates the opportunity loss beyond $K_a$ but up to $K_d$ for the protection buyer. In exchange, the unit premium $\xi$ applied to the remaining protected amount is paid to the protection seller by the protection buyer, where the value of $\xi$ is set in such a way that the no-arbitrage condition of the credit derivative market is satisfied; i.e., the expected profit without CDO would be equal to the expected profit with CDO including the premium payment. Consequently, the expected profit is indifferent about whether the CDO scheme is applied.

In the literature, the CDO has been studied largely from the perspective of how to deal with possible dependencies among defaults. See, for example, Li [9], Duffie and Garlean [3], Schonbucher and Schubert [11], Takada and Sumita [12], Takada, Sumita, and Takahashi [14], and Park [10]. With regard to computing the risk-neutral unit premium, see, for example, Lando [8], Kock, Kraft, and Steffensen [7], and Takada, Sumita, and Takahashi [13].

In order to examine the essential structure of the CDO in a neutral manner, we stay away from the problem of controlling financial risks and apply the CDO approach to the classical newsboy problem (NBP). The classical NBP is concerned with how to determine the optimal order quantity of a product whose value diminishes over one period, so as to maximize the expected profit, given the distribution of the stochastic demand. Instead of maximizing the expected profit, one often deals with the loss function, which can be expressed as the difference between the maximum possible profit and the actual profit, given the order quantity $Q$. This loss function consists of the loss due to the reduced residual value when the order quantity is larger than the actual demand and the opportunity loss when the order quantity is less than the actual demand. Clearly, maximizing the expected profit is equivalent to minimizing the expected loss function. The reader is referred to Khouja [6] for further details.

Recently, the classical NBP has been analyzed from the perspective of the value at risk (VaR) problem by Gotoh and Takano [4] and Chi and Choi [2]. This problem was revisited by Isogai, Ohashi, and Sumita [5] in a slightly different context, where the optimal order quantity $Q_{\text{NBP}}^{**}$ would be determined so as to minimize the probability $\eta_{\text{NBP}}^{**}$ of the profit falling below $v_0$ subject to the constraint that the expected profit stays above $v_1$. The distribution function $W_{\text{NBP}}(Q, x)$ of the profit without CDO, denoted by $PR_{\text{NBP}}(Q, D)$, was derived.
explicitly. In particular, when the stochastic demand is uniformly distributed, both $Q^*_{NBP}$ and $\eta^*_{NBP}$ were obtained explicitly in a closed form. The VaR problem for NBP with CDO was next formulated with $Q^*_{CDO}$ and $\eta^*_{CDO}$, and the distribution function $W_{CDO}(Q,x)$ of the profit with CDO, denoted by $PR_{CDO}(Q,D)$, was obtained. For the case of the uniformly distributed stochastic demand, numerical results were exhibited, demonstrating under what conditions the CDO scheme would be likely to be effective in that $\eta^*_{CDO} < \eta^*_{NBP}$; that is, the optimal solution for the VaR problem with CDO is better than that without CDO.

This paper is a sequel to the original paper, where some properties of the CDO scheme applied to NBP are established anew. More specifically, $W_{CDO}(Q,x)$ is re-expressed as a function of $Q$, rather than as a function of $x$ as in the original paper. This in turn enables one to establish conditions under which a stochastic ordering between $PR_{NBP}(Q,D)$ and $PR_{CDO}(Q,D)$ would be present; that is, stochastically $PR_{CDO}(Q,D)$ is dominated by $PR_{NBP}(Q,D)$ or dominates $PR_{NBP}(Q,D)$. These conditions are also sufficient to ensure $\eta^*_{CDO} \geq \eta^*_{NBP}$ for the former case and $\eta^*_{CDO} \leq \eta^*_{NBP}$ for the latter case. Furthermore, the VaR problem for NBP without CDO is analyzed for the case of the exponentially distributed stochastic demand. Based on these results, numerical experiments are presented, illustrating the above sufficient conditions and further providing rules of thumb for $\eta^*_{CDO} \leq \eta^*_{NBP}$ to hold; that is, one can expect the merits of the CDO scheme for the VaR problem with the exponentially distributed stochastic demand.

The structure of this paper is as follows. In Section 2, a succinct summary of the classical NBP and the associated VaR problem with CDO is provided from Isogai, Ohashi, and Sumita [5]. The distribution function $W_{CDO}(Q,x)$ of $PR_{CDO}(Q,D)$ is then re-expressed as a function of $Q$ in Section 3. Furthermore, sufficient conditions are established under which a stochastic ordering would be present between $PR_{NBP}(Q,D)$ and $PR_{CDO}(Q,D)$. It is shown that these conditions are also sufficient to ensure $\eta^*_{CDO} \geq \eta^*_{NBP}$ (or $\eta^*_{CDO} \leq \eta^*_{NBP}$). Section 4 is devoted to the detailed analysis of the VaR problem of NBP without CDO for the case of the exponentially distributed demand. Numerical results are given in Section 5, demonstrating that the merits of the CDO approach would exist under certain conditions for the exponential case. Finally, some concluding remarks are given in Section 6.
2. Classical Newsboy Problem and the Associated VaR Problem with CDO

We consider a product whose value drops substantially after a fixed point in time, say \( \tau \). The demand for the product over the period \([0, \tau]\) is given as a non-negative random variable \( D \). Throughout the paper, it is assumed that the distribution function of \( D \) is absolutely continuous with \( F_D(x) \equiv \mathbb{P}[D \leq x] - \int_x^\infty f_D(y) \, dy \) having the mean \( \mu_D = \mathbb{E}[D] \). The corresponding survival function is given by

\[
F_D(x) \equiv \mathbb{P}[D > x] = 1 - F_D(x) = \int_x^\infty f_D(y) \, dy.
\]

Let \( \tilde{c} \) and \( \tilde{p} \) be the procurement cost and the sales price per one product, respectively. Given that the order quantity is \( Q \), if \( Q \leq D \), each unsold product has the residual value \( \tilde{r} \). It is natural to assume that

\[
0 < \tilde{r} < \tilde{c} < \tilde{p}.
\]

If \( D > Q \), each of the lost opportunities would cost \( \tilde{s} \). Assuming that the payment would be made and the revenue would be received at time \( \tau \), the profit \( \tilde{P}_{NBP}(Q, D) \) can then be described as

\[
\tilde{P}_{NBP}(Q, D) = (\tilde{p} - \tilde{c})Q - (\tilde{p} - \tilde{r})[Q - D]^+ - \tilde{s}[D - Q]^+,
\]

(2)

where we define \([a]^+ = \max\{a, 0\}\). Let the distribution function and the expectation of \( \tilde{P}_{NBP}(Q, D) \) be denoted by

\[
\tilde{W}_{NBP}(Q, x) \equiv \mathbb{P}[\tilde{P}_{NBP}(Q, D) \leq x]; \quad \tilde{\pi}_{NBP}(Q) \equiv \mathbb{E}[\tilde{P}_{NBP}(Q, D)].
\]

(3)

The classical NBP is then to determine the optimal order quantity \( Q^{*}_{NBP} \) so as to maximize \( \tilde{\pi}_{NBP}(Q) \). For notational convenience, we write

\[
Q^{*}_{NBP} = \arg \max_Q \tilde{\pi}_{NBP}(Q).
\]

(4)
From (2), the maximum profit that one can expect is \((\bar{p} - \bar{c}) \times D\), which occurs if \(Q\) happens to be \(D\). The difference between this maximum profit and the actual profit may then be interpreted as the opportunity loss. More formally, we define

\[
\tilde{\iota}_{NBP}(Q, D) \overset{\text{def}}{=} (\bar{p} - \bar{c})D - \tilde{P}R_{NBP}(Q, D).
\] (5)

Let \(\tilde{c}_O\) and \(\tilde{c}_U\) be defined by

\[
\tilde{c}_O \overset{\text{def}}{=} \bar{c} - \bar{r} ; \quad \tilde{c}_U \overset{\text{def}}{=} \bar{p} - \bar{c} + \bar{s}.
\] (6)

One sees from (2) and (5) that

\[
\tilde{\iota}_{NBP}(Q, D) = \tilde{c}_O [Q - D]^+ + \tilde{c}_U [D - Q]^+.
\] (7)

Let the expectation of \(\tilde{\iota}_{NBP}(Q, D)\) be denoted by

\[
\mu_{\tilde{\iota}_{NBP}}(Q) = E[\tilde{\iota}_{NBP}(Q, D)].
\] (8)

It can be readily seen from (3) through (8) that maximizing \(\tilde{\pi}_{NBP}(Q)\) is equivalent to minimizing \(\mu_{\tilde{\iota}_{NBP}}(Q)\). It then follows that

\[
Q_{NBP}^* = \arg \min_Q \mu_{\tilde{\iota}_{NBP}}(Q).
\] (9)

From (7), it can be shown that

\[
\tilde{H}_{\tilde{\iota}_{NBP}}(Q, x) = P[\tilde{\iota}_{NBP}(Q, D) > x] = F_D(Q - \frac{x}{\tilde{c}_O}) + \tilde{F}_D(Q + \frac{x}{\tilde{c}_U}).
\] (10)

Since \(\mu_{\tilde{\iota}_{NBP}}(Q) = \int \tilde{H}_{\tilde{\iota}_{NBP}}(Q, x)dx\), it then follows that

\[
\mu_{\tilde{\iota}_{NBP}}(Q) = \tilde{c}_O \int_0^Q F_D(x)dx + \tilde{c}_U \int_Q^\infty \tilde{F}_D(x)dx.
\] (11)
It has been shown that $\mu_{\text{NBP}} (Q)$ is strictly convex in $Q$ and has the unique minimum point $Q^*_{\text{NBP}}$ given by

$$Q^*_{\text{NBP}} = F_D^{-1} \left( \frac{\tilde{c}_U}{\tilde{c}_O + \tilde{c}_U} \right).$$

(12)

The reader is referred to Khouja [6] for further details.

For incorporating the one-term CDO approach in the context of the NBP, it is necessary to convert the monetary values evaluated at time $\tau$, where such values are highlighted by ~ in the above discussions, into the corresponding present values. This can be accomplished by discounting the monetary values evaluated at time $\tau$ by $e^{-r_f \tau}$ where $r_f$ is the risk-free rate. The present value of a monetary value evaluated at time $\tau$ is denoted by dropping ~ in the notation. One can confirm the following conversions.

$$p = e^{-r_f \tau} \tilde{p}; \ c = e^{-r_f \tau} \tilde{c}; \ r = e^{-r_f \tau} \tilde{r}; \ s = e^{-r_f \tau} \tilde{s}; \ PR_{\text{NBP}}(Q, D) = e^{-r_f \tau} \tilde{P}R_{\text{NBP}}(Q, D);$$
$$W_{\text{NBP}}(Q, x) = e^{-r_f \tau} \tilde{W}_{\text{NBP}}(Q, x) \pi_{\text{NBP}}(Q) = e^{-r_f \tau} \tilde{W}_{\text{NBP}}(Q);$$
$$l_{\text{NBP}}(Q, D) = e^{-r_f \tau} \tilde{l}_{\text{NBP}}(Q, D); \ c_O = e^{-r_f \tau} \tilde{c}_O; \ c_U = e^{-r_f \tau} \tilde{c}_U.$$

(13)

From (13), it can be readily seen that $\pi_{\text{NBP}} (Q)$ achieves the maximum also at $Q^*_{\text{NBP}}$ and one has

$$Q^*_NBP = \arg \max_Q \pi_{\text{NBP}} (Q) = F_D^{-1} \left( \frac{\tilde{c}_U}{\tilde{c}_O + \tilde{c}_U} \right).$$

(14)

It should be noted from (3), (5), and (13) that

$$\pi_{\text{NBP}} (Q) = (p - c) \mu_D - \mu_{\text{NBP}} (Q),$$

(15)

where $\mu_{\text{NBP}} (Q)$ can be obtained from (11) and (13) as

$$\mu_{\text{NBP}} (Q) = (c_O + c_U) \int_0^Q F_D(x) dx + c_U (\mu_D - Q).$$

(16)
The next theorem provides a necessary and sufficient condition for the maximum expected profit \( \pi_{NBP}(Q^*) \) to be positive.

**Theorem 2.1** (Isogai, Ohashi, and Sumita [5])

\[
\pi_{NBP}(Q^*) > 0 \text{ if and only if } \frac{s}{c_o + c_U} < \frac{1}{\mu_D} \int_0^{Q^{sup}} s dF_D(x).
\]

Throughout the paper, we assume that the condition of Theorem 2.1 is satisfied.

Recently the classical NBP has been analyzed from the perspective of a conditional VaR problem by Gotoh and Takano [4]. In this paper, as in Isogai, Ohashi, and Sumita [5], the following VaR problem, which is different from that of Gotoh and Takano [4], is considered.

\[
[\text{VaR-NBP}] \quad \min_Q \eta_{NBP} \quad \text{subject to} \quad W_{NBP}(Q, v_0) \leq \eta_{NBP} ; \quad \pi_{NBP}(Q) \geq v_1
\]

From (15), the strict convexity of \( \mu_{l,NBP}(Q) \) implies the strict concavity of \( \pi_{NBP}(Q) \). Hence, there exist \( Q_{v_1,L} \) and \( Q_{v_1,R} \) such that the feasible region \( FR(v_1) \) for [VaR-NBP] can be written as

\[
FR(v_1) = \{ Q : \pi_{NBP}(Q) \geq v_1 \} = \{ Q : Q_{v_1,L} \leq Q \leq Q_{v_1,R} \}.
\]  

(17)

In order to apply the CDO scheme to the classical NBP, we let the loss function \( \tilde{L}_{NBP}(Q, D) \) in (7) replace the credit risk in the original CDO context. More specifically, given a tranche consisting of a pair of an attachment point \( K_a \) and a detachment point \( K_d \), we define \( \tilde{L}_{[K_a, K_d]}(\tau) \) by

\[
\tilde{L}_{[K_a, K_d]}(\tau) \overset{\text{def}}{=} \begin{cases} 
0 & \text{if } \tilde{L}_{NBP}(\tau) \in [0, K_a] \\
\tilde{L}_{NBP}(\tau) - K_a & \text{if } \tilde{L}_{NBP}(\tau) \in [K_a, K_d] \\
K_d - K_a & \text{if } \tilde{L}_{NBP}(\tau) \in [K_d, \infty]
\end{cases}
\]

where \( \tilde{L}_{[K_a, K_d]}(\tau) \) is the payment paid to the retailer issuing the CDO by the tranche investor buying the CDO at time \( \tau \). After a little algebra, as given in
Theorem 5.1 of Isogai, Ohashi, and Sumita [5], one finds that 

$$\mu_{\tilde{L}_{[K_a,K_d]}}(Q,D) = \mathbb{E}[\tilde{L}_{[K_a,K_d]}(Q,D)]$$

is given by

$$\mu_{\tilde{L}_{[K_a,K_d]}}(Q) = \int_{K_a}^{K_d} \left\{ F_D(Q - \frac{x}{\theta}) + F_D(Q + \frac{x}{\theta}) \right\} dx . \tag{18}$$

The risk neutral CDO premium $\xi$ is then given by

$$\xi(Q) = \frac{e^{-rT} \mu_{\tilde{L}_{[K_a,K_d]}}(Q)}{K_d - K_a} , \tag{19}$$

where we write $\xi(Q)$ in place of $\xi$ to emphasize that $\xi$ is a function of $Q$.

Let $PR_{CDO}(Q,D)$ and $PR_{NBP}(Q,D)$ be the present value of the profit with CDO and that without CDO, respectively. From (19), the present value of the amount to be paid to the tranche investor by the retailer is given by

$$\xi(Q) \times (K_d - K_a) = e^{-rT} \mu_{\tilde{L}_{[K_a,K_d]}}(Q) .$$

It then follows that

$$PR_{CDO}(Q,D) = PR_{NBP}(Q,D) - e^{-rT} \mu_{\tilde{L}_{[K_a,K_d]}}(Q) + e^{-rT} \tilde{L}_{[K_a,K_d]}(Q,D) . \tag{20}$$

In parallel with (3), we define

$$W_{CDO}(Q,x) \overset{\text{def}}{=} P[PR_{CDO}(Q,D) \leq x] ; \quad \pi_{CDO}(Q) \overset{\text{def}}{=} \mathbb{E}[PR_{CDO}(Q,D)] . \tag{21}$$

The VaR problem with CDO can now be formulated as follows.

$$\min_{Q} \eta_{CDO} \quad \text{subject to} \quad W_{CDO}(Q,v_0) \leq \eta_{CDO} ; \quad \pi_{CDO}(Q) \geq v_1 \tag{22}$$
In the next section, we express \( W_{cdo}(Q, x) \) as a function of \( Q \) and establish sufficient conditions under which a stochastic ordering would be present between \( PR_{NBP}(Q, D) \) and \( PR_{cdo}(Q, D) \). These conditions are also sufficient for the CDO scheme applied to NBP to be effective (or not effective).

3. **SUFFICIENT CONDITIONS FOR THE CDO SCHEME APPLIED TO NBP TO BE EFFECTIVE OR NOT EFFECTIVE**

The next two theorems in the previous paper are relevant to the discussions in this section. For notational convenience, the following functions are introduced.

\[
\begin{align*}
\xi_+(Q, x) &\equiv \frac{c_1 Q - x}{s}, \quad \xi_-(Q, x) \equiv \frac{c_2 Q + x}{p - r}, \\
\zeta_1(Q, x) &= \xi_-(Q, x) + \frac{e^{-r f T} \mu_{L|K_a K_d]}(Q)}{p - r}, \\
\zeta_2(Q, x) &= \xi_+(Q, x) - \frac{e^{-r f T} \mu_{L|K_a K_d]}(Q)}{s}, \\
\zeta_3(Q, x) &= \frac{x + e^{-r f T} \mu_{L|K_a K_d]}(Q) + e^{-r f T} K_a}{p - c}, \\
\zeta_4(Q, x) &= \frac{x + e^{-r f T} \mu_{L|K_a K_d]}(Q) + e^{-r f T} K_a}{p - c}, \\
\zeta_5(Q, x) &= \xi_-(Q, x) + \frac{e^{-r f T} \mu_{L|K_a K_d]}(Q) - e^{-r f T} (K_d - K_a)}{p - r}, \\
\zeta_6(Q, x) &= \xi_+(Q, x) - \frac{e^{-r f T} \mu_{L|K_a K_d]}(Q) - e^{-r f T} (K_d - K_a)}{s}.
\end{align*}
\]

**Theorem 3.1** (Isogai, Ohashi, and Sumita [5])

Let \( W_{NBP}(Q, x) = P[PR_{NBP}(Q, D) \leq x] \). One then has

\[
W_{NBP}(Q, x) = \begin{cases} 
F_D(\xi_-(Q, x)) + F_D(\xi_+(Q, x)) & \text{if } x \leq (p - c)Q, \\
1 & \text{otherwise}
\end{cases}
\]
Theorem 3.2  (Isogai, Ohashi, and Sumita [5])

Let \( W_{\text{CDO}}(Q, x) = P\left[ PR_{\text{CDO}}(Q, D) \leq x \right] \). One then has

\[
W_{\text{CDO}}(Q, x) = \sum_{i=1}^{6} G_i(Q, x),
\]

where \( G_i(Q, x) \) for \( i = 1, \cdots, 6 \), are given as follows.

\[
G_1(Q, x) = \begin{cases} 
0 & \text{if } x \in (-\infty, x_{1:a}) \\
F_D(\zeta_1(Q, x)) - F_D(Q - \frac{K_a}{C_D}) & \text{if } x \in [x_{1:a}, x_{1:Q}] \\
F_D(Q) - F_D(Q - \frac{K_a}{C_D}) & \text{if } x \in (x_{1:Q}, \infty)
\end{cases}
\]

where

\[
x_{1:Q} = (p - c)Q - e^{-r'T\mu}L[K_a, K_d](Q), \quad x_{1:a} = \begin{cases} 
x_{1:Q} - (p - r)\frac{K_a}{C_D} & \text{if } \frac{K_a}{C_D} \leq Q \\
x_{1:Q} - (p - r)Q & \text{if } Q < \frac{K_a}{C_D}
\end{cases}
\]

\[
G_2(Q, x) = \begin{cases} 
0 & \text{if } x \in (-\infty, x_{2:a}) \\
F_D(Q + \frac{K_a}{C_D}) - F_D(\zeta_2(Q, x)) & \text{if } x \in [x_{2:a}, x_{2:Q}] \\
F_D(Q + \frac{K_a}{C_D}) - F_D(Q) & \text{if } x \in (x_{2:Q}, \infty)
\end{cases}
\]

where

\[
x_{2:Q} = x_{1:Q}, \quad x_{2:a} = x_{1:Q} - \frac{sK_a}{c_U}.
\]

\[
G_3(Q, x) = \begin{cases} 
0 & \text{if } x \in (-\infty, x_{3:d}) \\
F_D(\zeta_3(Q, x)) - F_D(Q - \frac{K_a}{C_D}) & \text{if } x \in [x_{3:d}, x_{3:a}] \\
F_D(Q - \frac{K_a}{C_D}) - F_D(Q - \frac{K_a}{C_D}) & \text{if } x \in (x_{3:a}, \infty)
\end{cases}
\]

where

\[
x_{3:d} = \begin{cases} 
x_{1:Q} - e^{-r'T\mu}[K_a - (p - c)\frac{K_a}{C_D}] & \text{if } \frac{K_a}{C_D} \leq Q \\
x_{1:Q} - (p - c)Q - e^{-r'T\mu}K_a & \text{if } Q < \frac{K_a}{C_D}
\end{cases}
\]

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\[ x_{3:a} = \begin{cases} x_{1:Q} - e^{-r\tau}K_a - (p - c)\frac{K_a}{\xi} & \text{if } \frac{K_a}{\xi} \leq Q \\ x_{1:Q} - (p-c)Q - e^{-r\tau}K_a & \text{if } Q < \frac{K_a}{\xi} \end{cases}, \]

\[ G_4(Q, x) = \begin{cases} 0 & \text{if } x \in (-\infty, x_{4:a}) \\ F_D(\zeta_4(Q, x)) - F_D(Q + \frac{K_a}{\xi}) & \text{if } x \in [x_{4:a}, x_{4:d}] \\ F_D(Q + \frac{K_a}{\xi}) - F_D(Q + \frac{K_a}{\xi}) & \text{if } x \in (x_{4:d}, \infty) \end{cases}, \]

where

\[ x_{4:a} = x_{1:Q} - s\frac{K_a}{\xi}, \quad x_{4:d} = x_{1:Q} - e^{-r\tau}K_a + (p-c)\frac{K_d}{\xi}. \]

\[ G_5(Q, x) = \begin{cases} F_D(\zeta_5(Q, x)) & \text{if } x \in (-\infty, x_{5:d}) \\ F_D(Q - \frac{K_d}{\xi}) & \text{if } x \in (x_{5:d}, \infty) \end{cases}, \]

where

\[ x_{5:d} = \begin{cases} x_{1:Q} + \frac{(c-p)K_d}{\xi} - e^{-r\tau}K_a & \text{if } \frac{K_d}{\xi} \leq Q \\ x_{1:Q} - (p-r)Q + e^{-r\tau}(K_d - K_a) & \text{if } Q < \frac{K_d}{\xi} \end{cases}. \]

\[ G_6(Q, x) = \begin{cases} \bar{F}_D(\zeta_6(Q, x)) & \text{if } x \in (-\infty, x_{6:d}) \\ \bar{F}_D(Q + \frac{K_d}{\xi}) & \text{if } x \in (x_{6:d}, \infty) \end{cases}, \]

where

\[ x_{6:d} = x_{1:Q} - e^{-r\tau}K_a + (p-c)\frac{K_d}{\xi}. \]

The expression of \( W_{\text{CDO}}(Q, x) \) in Theorem 3.2 is inconvenient to study \( W_{\text{CDO}}(Q, v_0) \) as a function of \( Q \), which is needed to solve the VaR problem with CDO in (22). When \( v_0 > 0 \), \( W_{\text{CDO}}(Q, v_0) \) can be re-expressed as shown in the following theorem. The proof is lengthy, laborious, and more or less algebraic, and is omitted here.
Theorem 3.3

Let $x \geq 0$ and define

\[
q_1(x) = \frac{e^{-r x} \mu [K_a, K_d](Q)}{p - c s K_a} ; \quad q_2(x) = q_1(x) + \left( \frac{K_a}{\bar{p} - \bar{c}} - \frac{K_d}{\bar{c} U} \right) ; \\
q_3(x) = q_1(x) + \frac{K_a}{(p - c) \bar{c} U} ; \quad q_4(x) = q_1(x) + \frac{K_a (p - r)}{\bar{c} U (p - c)} ; \\
q_5(x) = q_1(x) + \left( \frac{K_a}{\bar{p} - \bar{c}} + \frac{K_d}{\bar{c} O} \right) .
\]  

The following three statements then hold.

1. \[ \max \{ q_1(x), q_2(x) \} < q_3(x) < q_4(x) < q_5(x) \] and \[ \max \{ q_1(x), q_2(x) \} = \begin{cases} q_2(x) & \text{if } \frac{K_d}{K_a} < \frac{p - c + s}{p - c} \\ q_1(x) & \text{if } \frac{K_d}{K_a} > \frac{p - c + s}{p - c} \end{cases} \]

2. If \( \frac{K_d}{K_a} < \frac{p - c + s}{p - c} \) then

\[
W_{CDO} (Q, x) = \begin{cases} 1 & \text{if } Q \leq q_1(x) \\ F_D(\zeta_1(Q, x)) + F_D(\zeta_2(Q, x)) & \text{if } q_1(x) \leq Q \leq q_2(x) \\ F_D(\zeta_1(Q, x)) + F_D(\zeta_2(Q, x)) + F_D(\zeta_3(Q, x)) + F_D(\zeta_6(Q, x)) & \text{if } q_2(x) \leq Q \leq q_3(x) \end{cases} .
\]

3. If \( \frac{K_d}{K_a} \geq \frac{p - c + s}{p - c} \), then

\[
W_{CDO} (Q, x) = \begin{cases} 1 & \text{if } Q \leq q_2(x) \\ F_D(\zeta_1(Q, x)) + F_D(\zeta_6(Q, x)) & \text{if } q_2(x) \leq Q \leq q_1(x) \\ F_D(\zeta_1(Q, x)) + F_D(\zeta_2(Q, x)) + F_D(\zeta_4(Q, x)) + F_D(\zeta_6(Q, x)) & \text{if } q_1(x) \leq Q \leq q_3(x) \end{cases} .
\]
We are now in a position to establish sufficient conditions under which a stochastic ordering would be present between $PR_{NBP}(Q,D)$ and $PR_{CDO}(Q,D)$. We recall that a non-negative random variable $X$ is stochastically larger than another non-negative random variable $Y$ if and only if $F_X(x) \geq F_Y(x)$ for all $x \geq 0$, where $F_X(x) = P[X > x]$ is the survival function of $X$. This ordering is often denoted by $X \succ_{ST} Y$. In this paper, we extend this concept to arbitrary random variables. More specifically, we define that a random variable $X$ is stochastically larger than another random variable $Y$ on $[0, \infty)$ if and only if $F_X(x) \geq F_Y(x)$ for all $x \geq 0$. This ordering is denoted by $X \succ_{ST+} Y$.

**Theorem 3.4**

Let $q_1(x)$, $q_2(x)$ and $q_5(x)$ be as in (30). The following two statements hold true for $x \geq 0$.

(a) If $\frac{K_d}{K_a} < \frac{p-c+s}{p-c}$ and $q_1(x) \leq Q \leq q_2(x)$, then

$$PR_{NBP}(Q,D) \succ_{ST+} PR_{CDO}(Q,D).$$

(b) If $Q \geq q_5(x)$ and $\mu_{\tilde{L}(K_a, K_d)}(Q) \leq K_d - K_a$, then

$$PR_{NBP}(Q,D) \prec_{ST+} PR_{CDO}(Q,D).$$

**Proof**

We note from Theorems 3.1 and 3.3(b) together with (26) and (29) that, for $x \geq 0$,

$$W_{NBP}(Q,x) - W_{CDO}(Q,x)$$

$$= \left\{F_D(\xi_-(Q,x)) - F_D\left(\xi_-(Q,x) + \frac{e^{-\tau T} \mu_{\tilde{L}(K_a, K_d)}(Q)}{p-\tau}\right)\right\}$$

$$+ \left\{F_D(\xi_+(Q,x)) - F_D\left(\xi_+(Q,x) - \frac{e^{-\tau T} \mu_{\tilde{L}(K_a, K_d)}(Q)}{s}\right)\right\}.$$
Since \( F_D(x) \) is monotonically increasing and \( \bar{F}_D(x) \) is monotonically decreasing, each of the two terms on the right hand side of the above expression is negative, and \( W_{NBP}(Q,v_0) \leq W_{CDO}(Q,v_0) \), proving (a). For part (b), we similarly observe that
\[
W_{NBP}(Q,x) - W_{CDO}(Q,x) = \left\{ F_D(\xi_-(Q,x)) - F_D \left( \xi_-(Q,x) - \frac{e^{-\tau f}(K_d - K_a) - e^{-\tau f} \mu_{\bar{E}[\xi]}(Q)}{p - r} \right) \right\} \\
+ \left\{ \bar{F}_D(\xi_+(Q,x)) - \bar{F}_D \left( \xi_+(Q,x) + \frac{e^{-\tau f}(K_d - K_a) - e^{-\tau f} \mu_{\bar{E}[\xi]}(Q)}{s} \right) \right\}.
\]
Under the condition \( \mu_{\bar{E}[\xi]}(Q) \leq K_d - K_a \), the monotonicity of \( F_D(x) \) and that of \( \bar{F}_D(x) \) imply that \( W_{NBP}(Q,v_0) \geq W_{CDO}(Q,v_0) \), completing the proof.

We next show that Theorem 3.4 also provides sufficient conditions for
\[
\eta_{CDO}^{**} \geq \eta_{NBP}^{**} \quad \text{or} \quad \eta_{CDO}^{**} \leq \eta_{NBP}^{**}.
\]

**Theorem 3.5**

Let \( Q_{v,L} \) and \( Q_{v,R} \) be as in (17). Then the following two statements hold.

(a) If \( \frac{K_d}{K_a} < \frac{p - c + s}{p - c} \) and \( Q_{v,R} \leq q_L(v_0) \), then \( \eta_{NBP}^{**} \leq \eta_{CDO}^{**} \).

(b) If \( q_L(v_0) \leq Q_{v,L} \) and \( \mu_{\bar{E}[\xi]}(Q) \leq K_d - K_a \) for \( \forall Q \in FR(v_1) \), then
\[
\eta_{NBP}^{**} \geq \eta_{CDO}^{**}.
\]
Proof
Suppose $Q_{v,R} \leq q_l(v_0)$ . One sees, from Theorem 3.3(b) and (17), together with the conditions in (a), that $W_{CDO}(Q,v_0) = 1$ for $\forall Q \in FR(v_1)$ so that $1 = \eta_{CDO}^{**} \geq \eta_{NBP}^{**}$.

If $q_1(v_0) \leq Q_{v,R}$, it again follows that $q_1(v_0) \leq Q_{CDO}^{**} \leq Q_{v,R} \leq q_2(v_0)$ . Consequently, from Theorem 3.4(a), one has

$$\eta_{CDO}^{**} = W_{CDO}(Q_{CDO}^{**}, v_0) \geq W_{NBP}(Q_{CDO}^{**}, v_0) \geq W_{NBP}(Q_{NBP}^{**}, v_0) = \eta_{NBP}^{**},$$

proving (a). For part (b), if $q_5(v_0) \leq Q_{v,l}$ and $PR_{NBP}(Q,D) \prec ST + PR_{CDO}(Q,D)$ for $\forall Q \in FR(v_1)$, Theorem 3.4(b) implies that $\mu_{L[K_r, K_d]}(Q) \leq K_d - K_a$ for $\forall Q \in FR(v_1)$. It then follows that

$$\eta_{NBP}^{**} = W_{NBP}(Q_{NBP}^{**}, v_0) \geq W_{CDO}(Q_{NBP}^{**}, v_0) \geq W_{CDO}(Q_{CDO}^{**}, v_0) = \eta_{CDO}^{**},$$

completing the proof.

4. ANALYSIS OF THE VAR PROBLEM WITH EXPONENTIAL STOCHASTIC DEMAND

The purpose of this section is to analyze VaR-NBP with the exponentially distributed stochastic demand. This in turn provides a basis for comparing the optimal solution of VaR-NBP with that of VaR-NBP-CDO via numerical examples in Section 5.

Let $D$ be exponentially distributed with p.d.f. $f_D(x)$ defined by

$$f_D(x) = \lambda e^{-\lambda x}U(x),$$

where

$$U(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}.$$  \hfill (32)

The distribution function and the survival function of $D$ can be written respectively as
\[ F_D(x) = (1 - e^{-\lambda x})U(x) ; \tilde{F}_D(x) = 1 - U(x) + e^{-\lambda x}U(x). \] (33)

From (14), one finds that the optimal order quantity \( Q_{NBP}^* \) maximizing the expected profit is given by

\[ Q_{NBP}^* = \frac{1}{\lambda} \log \left( 1 + \frac{c_U}{c_O} \right). \] (34)

From (15) together with (16), it follows that

\[ \pi_{NBP}(Q) = -c_O Q - \left( \frac{c_O + c_U}{\lambda} \right)e^{-\lambda Q} + \frac{p - r}{\lambda}. \] (35)

The corresponding maximum expected profit is then obtained from (34) and (35) as

\[ \pi_{NBP}(Q_{NBP}^*) = -\frac{c_O}{\lambda} \log \left( 1 + \frac{c_U}{c_O} \right) + \frac{p - c}{\lambda}. \] (36)

Throughout this section, it is assumed that

\[ v_0 \leq (p - c)Q_{VI:R}. \] (37)

Under this condition, the survival function of \( PR_{NBP}(Q,D) \), denoted by \( W_{NBP}(Q,v_0) = P[PR_{NBP}(Q,D) > v_0] = 1 - W_{NBP}(Q,v_0) \), can be obtained from Theorem 3.1 as

\[ W_{NBP}(Q,v_0) = e^{-\lambda \xi_-(Q,v_0)} - e^{-\lambda \xi_+(Q,v_0)} = e^{-\lambda \xi_-(Q,v_0)} \left\{ 1 - e^{-\lambda (\xi_+(Q,v_0) - \xi_-(Q,v_0)))} \right\} = e^{-\lambda \xi_-(Q,v_0)} \left( 1 - e^{-\lambda \Delta(Q,v_0)} \right), \] (38)

where \( \Delta(Q,x) = \xi_+(Q,x) - \xi_-(Q,x) \). We are now in a position to prove the next theorem.
Theorem 4.1

Under the condition of (37), the optimal solution of VaR-NBP can be obtained as

\[
(Q_{\eta_{NBP}}^*, \eta_{NBP}^*) = \begin{cases} 
(Q_{\eta_1:L}, W_{NBP}(Q_{\eta_1:L}, v_0)) & \text{if } \hat{Q}_{\text{exp}}(v_0) < Q_{\eta_1:L} \\
(\hat{Q}_{\text{exp}}(v_0), W_{NBP}(\hat{Q}_{\text{exp}}(v_0), v_0)) & \text{if } Q_{\eta_1:L} \leq \hat{Q}_{\text{exp}}(v_0) \leq Q_{\eta_1:R} \\
(Q_{\eta_1:R}, W_{NBP}(Q_{\eta_1:R}, v_0)) & \text{if } \hat{Q}_{\text{exp}}(v_0) > Q_{\eta_1:R}
\end{cases}
\]

where

\[
\hat{Q}_{\text{exp}}(v_0) = \frac{1}{p - c} \left[ v_0 + \frac{p - r + s}{\lambda(p - r)s} \log \left( 1 + \frac{(p - r + s)(p - c)}{s\lambda} \right) \right].
\]

Proof

From (38), one sees that

\[
\log \bar{W}_{NBP}(Q, v_0) = -\lambda \xi -(Q, v_0) + \log(1 - e^{-\lambda \Delta Q, v_0}). \tag{39}
\]

By differentiating (39) with respect to \(Q\), it follows that

\[
\frac{\partial}{\partial Q} \log \bar{W}_{NBP}(Q, v_0) = -\frac{\lambda}{p - r} \left\{ c_Q - \frac{(p - r + s)(p - c)}{s(e^{\lambda \Delta Q, v_0}) - 1} \right\}, \tag{40}
\]

and

\[
\frac{\partial^2}{\partial Q^2} \log \bar{W}_{NBP}(Q, v_0) = -\frac{\lambda^2 e^{\lambda \Delta Q, v_0}(p - r + s)^2(p - c)^2}{(e^{\lambda \Delta Q, v_0} - 1)^2 (p - r)^2 s^2} < 0. \tag{41}
\]

Hence, \(\log \bar{W}_{NBP}(Q, v_0)\) is strictly concave in \(Q\) and so is \(\bar{W}_{NBP}(Q, v_0)\).

Accordingly, \(\bar{W}_{NBP}(Q, v_0)\) has the unique maximum at \(\hat{Q}_{\text{exp}}(v_0)\), satisfying

\[
\frac{\partial}{\partial Q} \log \bar{W}_{\text{exp}}(Q, v_0) \bigg|_{Q = \hat{Q}_{\text{exp}}(v_0)} = 0.
\]
This then implies that \( W_{NBP}(Q, v_0) \) takes the unique minimum at \( \hat{Q}_{exp}(v_0) \). From (40), it can be readily seen that
\[
\hat{Q}_{exp}(v_0) = \frac{1}{p-c} \left[ v_0 + \frac{p-r+s}{\lambda(p-r)s} \log \left\{ 1 + \frac{(p-r+s)(p-c)}{sc} \right\} \right].
\]

If \( \hat{Q}_{exp}(v_0) < Q_{v,L} \), then \( W_{NBP}(Q, v_0) \) is monotonically increasing in \( Q \in FR(v_i) \) and \(( Q^{**}_{NBP}, \eta^{**}_{NBP} ) = ( Q_{v,L}, W_{NBP}(Q_{v,L}, v_0) ) \). If \( Q_{v,L} \leq \hat{Q}_{exp}(v_0) \leq Q_{v,R} \), \( W_{NBP}(Q, v_0) \) clearly takes the minimum value at \( \hat{Q}_{exp}(v_0) \) and therefore \(( Q^{**}_{NBP}, \eta^{**}_{NBP} ) = ( \hat{Q}_{exp}(v_0), W_{NBP}(\hat{Q}_{exp}(v_0), v_0) ) \).

Finally, when \( \hat{Q}_{exp}(v_0) > Q_{v,R} \), \( W_{NBP}(Q, v_0) \) is monotonically decreasing in \( Q \in FR(v_i) \) so that \(( Q^{**}_{NBP}, \eta^{**}_{NBP} ) = ( Q_{v,R}, W_{NBP}(Q_{v,R}, v_0) ) \), completing the proof.

5. NUMERICAL RESULTS

In this section, we illustrate Theorem 3.5 through numerical examples. Furthermore, in order to explore the potential of the CDO scheme applied to NBP in a more general context, the optimal solution for VaR-NBP and that for VaR-NBP-CDO would be compared by altering the underline parameters \( v_1 \), \( v_0 \), and \( p \). The basic set of the parameter values is provided in Table 1, which will be used throughout this section unless specified otherwise.

| \( p \) | the unit sales price | 3 |
| \( c \) | the unit procurement cost | 1 |
| \( r \) | the unit residual value | 0.1 |
| \( s \) | the unit opportunity cost | 0.5 |
| \( \mu_D \) | the mean of the demand | 5000 |
| \( r_f \) | the risk free rate | 0.0001 |
| \( K_a \) | the attachment point | 500 |
| \( K_d \) | the detachment point | 600, 1000, 3000, 5000 |
The expected loss \( \mu_{\text{NBP}}(Q) \) and the expected profit \( \pi(Q) \) are first plotted in Figures 1 and 2, respectively. The former is strictly convex and the latter is strictly concave, as expected.

\[
\mu_{\text{NBP}}(Q)
\]

\[
\pi(Q)
\]

Figure 1. Expected Loss \( \mu_{\text{NBP}}(Q) \)

Figure 2. Expected Profit \( \pi(Q) \)
Figure 3 illustrates Theorem 5.3 (a) with $v_i = 4000$ and $v_0 = 14200$.

One sees from Table 1 that 
\[
\frac{K_d}{K_u} = 1.2 < 1.25 = \frac{p-c+s}{p-c}
\]
and $Q_{v_i} = 7111 \leq 7158 = q_2(v_0)$, and the conditions are satisfied.
We observe that \((Q_{\text{NBP}}^*, \eta_{\text{NBP}}^*) = (Q_{\nu^L-R}, W_{\text{NBP}}(Q_{\nu^L-R}, v_0)) = (7111, 0.997)\)
and \((Q_{\text{CDO}}^*, \eta_{\text{CDO}}^*) = (Q_{\nu^L-R}, W_{\text{CDO}}(Q_{\nu^L-R}, v_0)) = (7111, 1.000)\).

Theorem 5.3 (b) is illustrated numerically in Figure 4.

![Figure 4. Illustration of Theorem 3.5(b)](image)

We note that \(q_5(v_0) = 5578 \leq 6195 = Q_{\nu^L} \).

Figure 5 shows \(\mu_{l([K_x, K_y])}(Q)\) as a function of \(Q\), demonstrating \(\mu_{l([K_x, K_y])}(Q) \leq K_d - K_a\) so that the conditions for Theorem 5.3 (b) are satisfied. For this case, we observe that

\((Q_{\text{NBP}}^*, \eta_{\text{NBP}}^*) = (Q_{\nu^L}, W_{\text{NBP}}(Q_{\nu^L}, v_0)) = (6195, 0.411)\)

and \((Q_{\text{CDO}}^*, \eta_{\text{CDO}}^*) = (Q_{\nu^L}, W_{\text{CDO}}(Q_{\nu^L}, v_0)) = (6195, 0.392)\).
In Figure 6, $W_{NBP}(Q,2000)$ and $W_{CDO}(Q,2000)$ are plotted along with $\pi(Q)$, where $v_1$ is varied from 3000 and 3500 to 4000, while $K_d = 3000$ is fixed. One sees that the feasible region $FR(v_1)$ becomes narrower as the threshold, $v_1$, of the expected profit increases. Accordingly, both $\eta_{NBP}^{**}$ and $\eta_{CDO}^{**}$ become worse and increase as $v_1$ increases. It is worth noting that the CDO approach is effective only when $v_1$ becomes sufficiently large.

Similarly to Figure 6, $W_{NBP}(Q,v_0)$, $W_{CDO}(Q,v_0)$ and $\pi(Q)$ are depicted in Figure 7, where $v_0$ is varied from 1000 and 1500 to 2000, while $v_1$ is fixed at 3500 with $K_d = 3000$. It can be observed that it becomes more difficult to control the profit as the threshold $v_0$ becomes larger. While $\eta_{NBP}^{**} > \eta_{CDO}^{**}$ for $v_0 = 1000, 1500$, this inequality is reversed and the CDO approach becomes ineffective for $v_0 = 2000$. 

**Figure 5.** $\mu_{[K_a, K_d]}(Q)$

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International Journal of Business and Information
Figure 6  $\pi(Q)$ and $W_{NBP}(Q, 2000)$ vs. $W_{CDO}(Q, 2000)$  [ $K_d = 3000$ ]

$v_1 = 3000$  \hspace{1cm}  $v_1 = 3500$  \hspace{1cm}  $v_1 = 4000$
Analysis of the Collateralized Debt Obligation Scheme
Applied to the Newsboy Problem

Figure 7

$\pi(Q)$ and $W_{CDO}(Q, r_v)$ vs.

Table 1

$R = 3000$, $v = 3500$

$R = 2000$

$R = 1500$

$R = 1000$
In order to observe the impact of the price on the optimal solutions more closely, Figure 8 depicts $\eta_{NBP}^{**}$ and $\eta_{CDO}^{**}$ as functions of $p$, where $v_0 = 2000$, $v_1 = 3500$ and $K_d = 3000$ are fixed. The CDO approach becomes effective when $p$ becomes sufficiently large.

![Figure 8](image-url)

**Figure 8.** $\eta_{NBP}^{**}$ vs. $\eta_{CDO}^{**}$ [ $K_d = 3000$, $v_0 = 2000$, $v_1 = 3500$ ]

Finally, Figure 9 illustrates how $\eta_{NBP}^{**}$ and $\eta_{CDO}^{**}$ are impacted when $(v_0, v_1)$ and $K_d$ are changed, where the white areas represent the regions in which the CDO approach is effective. It can be observed that the CDO approach can be effective only when $v_1$ is sufficiently large for $K_d = 5000$. The area in which the CDO approach performs better shifts toward the lower side of $v_0$ and becomes larger as $K_d$ decreases.
6. CONCLUDING REMARKS

As a sequel to the original paper by the same authors, Isogai, Ohashi, and Sumita [5], this paper examines the effect of the CDO scheme applied to the classical NBP. The distribution function $W_{CDO}(Q,x)$ of $PR_{CDO}(Q,D)$ is re-expressed as a function of $Q$. Furthermore, sufficient conditions are established under which a stochastic ordering would be present between $PR_{NBP}(Q,D)$ and $PR_{CDO}(Q,D)$. It is shown that these conditions are also sufficient to ensure $\eta_{CDO}^{**} \geq \eta_{NBP}^{**}$ (or $\eta_{CDO}^{**} \leq \eta_{NBP}^{**}$). The VaR problem of NBP without CDO is analyzed in detail for the case of the exponentially distributed demand, deriving $Q_{NBP}^{**}$ and $\eta_{NBP}^{**}$ explicitly.

Extensive numerical experiments reveal that the overall effect of CDO is present when the underlying risk for the opportunity loss is rather large. More specifically, assuming that the stochastic demand $D$ is exponentially distributed, the CDO approach could become effective if

1. the expected profit should be held above a high level;
2. the probability of having a huge loss should be contained;
3. the price is very high; and
4. the detachment point $K_d$ should be held relatively low.
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Analysis of the Collateralized Debt Obligation Scheme
Applied to the Newsboy Problem


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